

The Missing Mass of the Milky Way Galaxy

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Abstract

A model is proposed in which cosmic ray protons flow radially through the galaxy. The resulting electric field energy creates a gravitational force, in addition to the conventional Newtonian force. The model yields a rotation curve that agrees in detail with the experimental curve. The total electric field energy is calculated. It is the missing mass of the galaxy.

1. The Electric Field.

The rotation curves of many galaxies have now been measured. They exhibit a characteristic flat velocity profile at points far from the galactic center. These observations cannot be understood in terms of conventional Newtonian theory, if visible mass is taken to be the sole source of gravitation. This apparent failure underlies the belief that a great deal of mass is missing from the cosmos.

Here, the argument is made that electric field energy contributes to the gravitational field of the galaxy. The electric field arises from cosmic ray protons that emanate from the central bulge of the galaxy. They stream radially through the core, through the halo, and eventually exit the galaxy.¹ These protons are confined only by the limiting speed of light. In a steady state, the amount of electric charge passing through each spherical surface is given by

$$\frac{dQ}{dt} = \rho v A = \rho c (4\pi r^2) \quad (1)$$

where r is the distance from the center. This yields the formula for charge density in the galaxy

$$\rho = \frac{\dot{Q}}{4\pi c r^2} = \frac{\chi}{r^2} \quad (2)$$

The electric field satisfies the equation $\nabla \cdot E = -\nabla^2 \phi = 4\pi\rho$. In the present case,

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\phi}{dr} \right) = -\frac{4\pi\chi}{r^2} \quad (3)$$

Integrate once to obtain

$$E = -\frac{d\phi}{dr} = \frac{4\pi\chi}{r} - \frac{k}{r^2} \quad (4)$$

where k is the constant of integration. If the central bulge (radius r_b) is taken to be electrically neutral, then $k = 4\pi\chi r_b$ and

$$E = \frac{4\pi\chi}{r} \left(1 - \frac{r_b}{r} \right) \quad (5)$$

¹Electrons exit the galaxy via the disk. Being less massive, they are subject to the magnetic field of the disk.

2. Gravity as $r \rightarrow \infty$.

In regions far from the galactic center, beyond the core and in the halo, electric field energy is the only source of gravitation. When expressed in terms of the energy density T_{00} , Newton's law reads

$$\nabla^2\psi = \frac{4\pi G}{c^2} T_{00} \quad (6)$$

The energy density of the electric field is $T_{00} = E^2/8\pi$, therefore

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = \frac{G}{2c^2} \left(\frac{4\pi\chi}{r} \right)^2 \left(1 - \frac{r_b}{r} \right)^2 \quad (7)$$

Integrate this equation to find

$$\frac{d\psi}{dr} = \frac{8\pi^2 \chi^2 G}{c^2} \left\{ \frac{1}{r} - \frac{r_b}{r^2} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K}{r^2} \quad (8)$$

The corresponding velocity curve is obtained by equating $-d\psi/dr$ to the centripetal acceleration $-v^2/r$

$$v^2 = \frac{8\pi^2 \chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K}{r} \quad (9)$$

In the limit $r \rightarrow \infty$, this yields the expression

$$v_\infty = \frac{2\pi\chi}{c} (2G)^{1/2} \quad (10)$$

As $r \rightarrow \infty$, the rotation curve gradually rises toward this asymptotic value.

3. Gravity in the Core and Halo.

In the core of the galaxy, outside the dense central bulge, the mass density is assumed to be a constant ρ_c . In this region, the gravitational field satisfies

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) = \frac{4\pi G}{c^2} \left(\rho_c c^2 + \frac{E^2}{8\pi} \right) \quad (11)$$

This equation integrates to

$$\frac{d\psi}{dr} = \frac{4}{3} \pi \rho_c G r + \frac{8\pi^2 \chi^2 G}{c^2} \left\{ \frac{1}{r} - \frac{r_b}{r^2} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} + \frac{K'}{r^2} \quad (12)$$

Near the periphery of the bulge, the electrical term is zero, and $d\psi/dr = GM_b/r_b^2$. Therefore, the constant of integration is

$$K' = GM_b \left(1 - \frac{\rho_c}{\rho_b} \right) \quad (13)$$

where $M_b = \frac{4}{3} \pi \rho_b r_b^3$. This yields the expression for velocities in the core

$$v^2 = \frac{GM_b}{r} \left(1 - \frac{\rho_c}{\rho_b} \right) + \frac{4}{3} \pi \rho_c G r^2 + \frac{8\pi^2 \chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} \quad (14)$$

The first term is a decreasing function of r , while the latter two terms are increasing. At the edge of the core, where $r = r_c$, formula (14) may be equated to (9). This determines the integration constant

$$K = G(M_b + M_c) \quad (15)$$

where $M_c = \frac{4}{3} \pi \rho_c (r_c^3 - r_b^3)$. It follows that velocities in the halo satisfy

$$v^2 = \frac{G(M_b + M_c)}{r} + \frac{8\pi^2 \chi^2 G}{c^2} \left\{ 1 - \frac{r_b}{r} \left[2 \ln \frac{r}{r_b} + \frac{r_b}{r} \right] \right\} \quad (16)$$

4. Numerical Results. [1, 2]

The experimental value of v_∞ is very uncertain. We choose $v_\infty = 240 \text{ km s}^{-1}$ in order to obtain good agreement in the core and halo. Formula (10) then gives the value of χ for our galaxy

$$\chi = 3.2 \times 10^{20} \text{ statvolts} \quad (17)$$

The mass M_b may be calculated from $GM_b/r_b = v_b^2$, using data from the rotation curve: $r_b = 600 \text{ parsecs (pc)}$ and $v_b = 260 \text{ km s}^{-1}$

$$M_b = 10^{10} \text{ solar masses} \quad (18)$$

The mass M_c is calculated by means of (16), using data for the Sun: $r_0 = 8.4 \text{ kpc}$ and $v_0 = 220 \text{ km s}^{-1}$

$$M_c = 1.5 \times 10^{10} \text{ solar masses} \quad (19)$$

Together with the core radius, $r_c = 4.0 \text{ kpc}$, this determines all parameters in formulae (14) and (16). They predict the following values for the rotation curve:

r	.6	1	2	3	4	6	8.4	12	24	48	∞
v	260	220	187	202	224	219	220	222	227	231	240

The fit with the experimental curve is remarkable.

The number density of protons may be calculated by setting $\rho = ne$ in formula (2). It varies from $n = 2 \times 10^{-13} \text{ cm}^{-3}$ near the bulge to $n = 2 \times 10^{-16} \text{ cm}^{-3}$ near the galactic fringe. This is much less than the density of cosmic rays that impinge upon Earth's atmosphere ($10^{-10} \text{ particles cm}^{-3}$). Nevertheless, a large number of protons leave the halo each second: set $Q = Ne$ in (1) to find

$$\frac{dN}{dt} = \frac{4\pi\chi c}{e} = 2.5 \times 10^{41} \text{ protons sec}^{-1} \quad (20)$$

[In a steady state, an equal number of electrons would leave the galaxy by way of the disk.]

The total energy of the electric field is given by

$$\int \frac{E^2}{8\pi} 4\pi r^2 dr = 8\pi^2 \chi^2 r_b \left(\frac{r}{r_b} - 2 \ln \frac{r}{r_b} - \frac{r_b}{r} \right) \quad (21)$$

This integral diverges linearly. If an arbitrary cutoff is introduced at $r = 60$ kpc, then the energy is nearly that of 10^{12} solar masses. This electric field energy is some 20 times greater than the visible mass energy in our galaxy. It is the ‘missing mass’ of the galaxy.

The potential difference is given by

$$\Delta\phi = - \int E dr = -4\pi\chi \left(\ln \frac{r}{r_b} + \frac{r_b}{r} - 1 \right) \quad (22)$$

Here, the divergence is logarithmic; with the cutoff, $\Delta\phi = -4 \times 10^{24}$ volts. If each proton in (20) were to escape with 10^{24} eV, then the entire energy content of the galaxy would be depleted in 10^4 years. Therefore, the vast majority of protons must attain an energy no greater than 10^{18} eV. Cosmic ray measurements show this to be the case.

5. Concluding Remarks.

The strength of the electric field (5) is not great. It reaches a maximum of $0.5 \text{ statvolts cm}^{-1}$ at $r = 2r_b$, then decreases to $0.1 \text{ statvolts cm}^{-1}$ near the galactic fringe. [Galactic magnetic fields are of the order 10^{-6} gauss. Thus, the radial electric force on a given proton is far greater than any magnetic force.] The energy density $E^2/8\pi$ is similar in magnitude to the mass energy density in the core ρ_cc^2 . The enormous total energy of the field (10^{12} solar masses) is due to its huge volume and not to its strength.

This electric field is fixed in the galaxy, sustained by the flow of highly energetic cosmic ray protons. These protons, emerging from the center of the galaxy, are spatially ordered, because the speed of light yields a $1/r^2$ steady state distribution of particles (2). This large scale flow of cosmic ray protons differs qualitatively from the random motion of ions that form the galactic plasma.

References.

1. B. Carroll and D. Ostlie, *An Introduction to Modern Astrophysics*, (Addison-Wesley, 1996) chapter 22.
2. V. Kong and G. Rainey, *The Disk Rotation of the Milky Way Galaxy*, www.csupomona.edu/~jis/1999/kong.pdf